Gravity field measurements using cold atoms with direct optical readout

A. B. Matsko, N. Yu, and L. Maleki
Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Drive, Pasadena, California 91109-8099

(Received 4 October 2002; published 28 April 2003)

We show theoretically that measuring an optimum observable of an electromagnetic wave propagating in an atomic medium moving freely in a gravitational field results in a sensitive detection of the acceleration of gravity and its gradient. The best achievable sensitivity of such a measurement is comparable with that of light-pulse atom interferometers based on measuring the atomic internal state. This optical technique is useful for nondestructive detection of ultracold atoms in atomic interferometers.

DOI: 10.1103/PhysRevA.67.043819 PACS number(s): 42.50.Gy, 42.50.Lc, 03.65.Ta, 42.62.Eh

I. INTRODUCTION

Pulsed-light atom interferometry has been successfully used for precision measurements of photon recoil, inertial acceleration, gravity gradient, and for other metrological applications [1–10]. In experiments based on this technique, laser light is used to manipulate cold atoms through interaction with both the internal and external atomic states. Any change resulting from the interaction with light and the atomic state is measured by probing the atomic energy level populations.

The state of an atom may also be studied by detecting the state of the photons. For instance, scattered light has been used for spatial observation of a Bose condensate, and dispersive nondestructive light scattering was used to observe the separation between the condensed and normal components of the Bose gas [11]. Probe light has also been used in the time-domain de Broglie wave interferometry [12], where the atomic population is observed by scattering an off-resonant traveling wave and subsequently measuring the light properties, using the heterodyne technique.

When light interacts with an atom, the atom changes its internal state, but the interacting light is also left with an altered state. Therefore, one can retrieve complementary information by measuring the state change of light, as well as the atoms. For instance, if the interaction is refractive and the atomic population distribution is not changed, then the interaction results in a modification of the phase of the light field. Such an interaction may also cause a birefringence. Further, if the atomic population distribution changes, the photon distribution of the light is affected as well. Such an interaction may result in a dichroism.

In this work we propose an optically based approach to the atom interferometry sensing problem that promises a similar sensitivity to the conventional pulsed-light atom interferometry, which detects atomic states. Our scheme is based on the detection of an observable of the probe light that interacts with the cold atoms. We show that the best sensitivity of this technique is achieved if one measures an *optimum observable*, which may be either a phase shift of the probe light, or light polarization, or the photon number, depending on the particular experimental conditions.

As is shown below, the first type of interaction mentioned above appears when two counterpropagating circularly polarized quasicontinuous electromagnetic waves interact with an atomic cloud. The electromagnetic waves are nearly resonant with an atomic transition having ground-state angular momentum F larger than the excited state $F' = F-1$. The resonant absorption is suppressed in this configuration due to the effect of coherent population trapping. The second type emerges from the interaction of short light pulses with the cloud, as in the usual atom interferometers [6]. In this case, the light is far detuned from the corresponding atomic transition to suppress the spontaneous emission. The population of atomic levels is then changed via the stimulated Raman process.

The amount of information in both atomic and electromagnetic channels varies depending on the photon number in detected probe wave, and the total number of atoms. At first glance it might seem that the photon number is the parameter of choice since an increase of the photon number may lead to an increase of the measurement sensitivity. It turns out, however, that the optimization of the measurements sensitivity with respect to light absorption results in an optimum photon number that is approximately equal to the number of atoms and, therefore, is very small. The measurement sensitivity scales inversely with the square root of the detected photon number, if the atomic cloud is initially prepared in an optimum coherent state and the absorption is suppressed.

This result may be understood if one notes that the interferometric measurement of the gravity field by optical means, as discussed in this paper, is based on the quantum nondemolition measurement of a projection of the momentum of the atom. This kind of measurements was recently proposed for two-level [13] and three-level Λ atoms [14]. The maximum sensitivity of such a measurement can be achieved for single photon scattering, as was pointed out by von Neumann for the Doppler measurement scheme of a mechanical momentum projection [15].

Concentrating on the observation of other properties of light, we note that a phase measurement or observation of an induced atomic medium birefringence may be performed directly or via polarization rotation. Polarization rotation measurement is an effective spectroscopic tool for measuring a change in the phase of light. Indeed, it allows a precise measurement of magnetic fields and fundamental constants. The sensitivity of the measurement is generally determined by the width of the resonance that is induced in a coherent atomic medium. Very narrow resonances (~ 1 Hz) achieved recently in atomic gas cells result in a sensitivity of 10^{-12} G in magnetic-field measurement [16]. This type of experiments are based on the energy shift of atomic Zeeman sublevels, which results in circular birefringence and leads to a polarization rotation of the probe light propagating through the medium.

Similarly, when a moving atom interacts with counterpropagating light having opposite circular polarization, the Doppler effect induces an effective frequency shift. This shift can change the detuning of light from the corresponding atomic transition in a manner similar to the Zeeman shift in the magnetometry application. Consequently, two counterpropagating circularly polarized waves, initially produced by a decomposition of a linearly polarized light and recombined after the interaction with a moving atomic ensemble, will produce a polarization rotation proportional to the atom velocity. An appropriate filtering procedure may be used to eliminate any initial velocity spread of the ensemble of atoms, and to retrieve information about the value of the acceleration of the atoms.

One may also directly detect a phase shift of the probe light propagating through a moving atomic cloud driven by a coupling field, instead of the polarization rotation measurement. This scheme is similar to a coherent magnetometer proposed in Ref. [17], and the sensitivity and properties of such a device are similar to those based on the polarization rotation.

To obtain a large change in the phase of light that propagates through a moving atomic medium, the atoms should possess a narrow spectral feature. Atoms of a Λ-type energy level configuration are convenient for such a scheme because narrow features can be readily obtained from two photon resonances. Generation of a slow group velocity of light and nonlinear resonant polarization rotation, for example, result from such an interaction.

Atomic fountains are widely used for precision measurements with pulsed light. When short coupling pulses are used in the experiments, the phase of the light pulse is hardly changed, in contrast to that of the quasi-cw light discussed above. We show that a detection of the photon number of the pulses used for the manipulation of the atomic state in the fountain may become a source of additional information about the number of atoms participating in the interaction, as well as the interferometer's signal itself. These optical methods of the atomic population measurements may relax some technical limitations of existing measurement techniques.

To achieve higher accuracy in fountain measurements the number of atoms participating in the interaction must be exactly determined. In conventional fountain experiments, the preparation uncertainty of the atomic clouds typically exceeds the quantum projection noise. Thus, most clocks and other fountain base atom interferometers approach shot-noise limited sensitivities via special normalization procedures.

To improve the sensitivity of measuring the atomic number, either fluorescence measurements or measurement of absorption of the probe radiation is used. For instance, fluorescence detection works quite well in a differentially pumped system used in the majority of fountain clocks. The sensitivity of the fluorescence detection may be hindered, however, because of the background noise due to the thermal atoms.

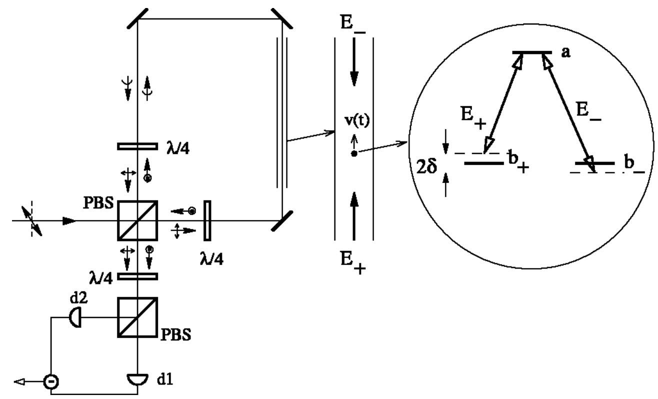

FIG. 1. Measurement scheme. A three-level atom interacts with two counterpropagating waves having the opposite circular polarizations. The atomic motion results in detuning of the fields from the corresponding atomic transitions in the atom's frame of reference. This detuning results in a change of the index of refraction for the waves that may be measured in the interferometric scheme.

The detection of light absorption imposes a restriction on the quality of the light source, which must have low intensity and frequency noise. Moreover, light absorption and consequent fluorescence result in heating of the atomic cloud.

An efficient method for low noise detection of cold atoms based on modulation transfer spectroscopy was proposed very recently [18], where the balanced detection technique was used to suppress laser induced detection noise. Our method is complimentary to the existing techniques. It has the advantages of the balanced modulation transfer spectroscopy and does not require introduction of substantial changes to the already existing atomic fountain setups. The measurement sensitivity of our method is particularly attractive when the atomic density is high and the total number of atoms in the cloud is large.

In the gravity gradiometer applications there is a distinct advantage for probing the light field directly. We show that with this scheme it is possible to read out the gravity gradient directly with a single measurement. This approach simplifies the experimental setup, but it may also further reduce the common-mode noise associated with the measurement of separate atomic ensembles in the two accelerometers of the gravity gradiometer.

The paper is organized as follows. In Secs. II and III we focus on the properties of polarization rotation accelerometer and gradiometer and show that the maximum sensitivity of the technique is nearly the same as that based on the light-pulse atom interferometer. In Sec. IV we describe the pulsed mode of the optical measurement in the fountain-based gradiometer.

II. MEASUREMENT SCHEME

Let us consider a Λ-type energy level scheme (see Fig. 1 inset) interacting with two resonant counterpropagating light waves with electric fields E_+ and E_-. The corresponding Rabi frequencies for the fields are $\Omega_\pm = \wp_\pm E_\pm/\hbar$, where $\wp_\pm$ are the dipole moments of the respective atomic transitions, which for the sake of simplicity we assume are equal ($\wp_\pm$

$=\wp$). The population radiative decay $2\gamma_r$ of the exited level $|a\rangle$ is given by the expression $\gamma_r = 4\omega^3\wp^2/(3\hbar c^3)$. The decay rate of the populations of ground state levels as well as the decay rate of low frequency atomic coherence γ_0 is determined by the atomic interaction time with the laser beam.

A possible configuration for the experimental setup is shown in Fig. 1. A linearly polarized wave is decomposed into two linearly (horizontally and vertically) polarized light waves via an input polarizing beam splitter (PBS) that is orientated at 45° with respect to the incoming light polarization. The linearly polarized light waves are transformed into circularly polarized waves via $\lambda/4$ wave plates placed after the beam splitter. After the interaction with the atom cloud, the waves propagate through the same $\lambda/4$ plates and become linearly polarized waves emerging through the second port of the input beam splitter.

The information about the atomic velocity is stored in the relative phase shift of the waves 2ϕ, proportional to the Doppler frequency shift δ. To measure it we first send the beam through another $\lambda/4$ plate rotated at 45° with respect to both polarizations, as in a polarization analyzer arrangement. The polarization rotation and the light intensity can be measured by the two photodetectors $d1$ and $d2$ at the second PBS output ports. The signals from these photodiodes are proportional to $(1/2)(P_+ + P_- \pm 2\sqrt{P_+P_-}\sin 2\phi)$, where P_+ and P_- are the powers of the circularly polarized components of the field. Therefore, the sum of the photocurrents yields the total output power, and the difference of the photocurrents gives the polarization rotation angle [19].

The polarization rotation angle is a linear function of the atomic velocity, $\phi(t) = a + bgt$, where a and b are some coefficients, the exact meaning of which will be explained in the following sections, g is the gravity acceleration, and t is the time. The value of a is unknown due to the initial uncertainty of the atomic momentum. By an appropriate usage of a filtering procedure, i.e., by measuring $\int G(t)\phi(t)dt$, it is possible to filter out the unknown parameter a and, hence, avoid the influence of uncertainty in the initial external degrees of freedom of the atomic state on the measurement sensitivity. This filtering procedure, discussed in detail in Sec. IV A, is similar to the measurement strategy described in Ref. [20].

If we place two atomic ensembles in the two different arms of the laser beam path, as shown in Fig. 2, the polarization rotations induced by the two moving media of the same velocity directions have opposite sign. For example, if the left atom ensemble has a δ_1 shift for E_+ and $-\delta_2$ for E_-, then the right atom ensemble has a $-\delta_1$ shift for E_+ and δ_2 for E_-. In such an arrangement, a single phase measurement gives the velocity difference between the two atomic ensembles, provided that they have the same average number of atoms. Since the two ensembles share the same laser system, the measurement gives the noninertial gravity gradient.

Again, an appropriate filter function will recover the acceleration difference of the two atomic ensembles. The polarization rotation in the scheme is equal to $\phi = a_1 - a_2 + [b_1 g(x_1) - b_2 g(x_2)]t$, where x_1 and x_2 are the spatial positions of the atomic clouds. The momentum uncertainty term $a_1 - a_2$ may be removed by filtering. If the average number of atoms in both clouds is the same, then $b_1 = b_2$ and the setup gives us information about $g(x_1) - g(x_2)$, i.e., the gradient of the gravity field. If the number of atoms in the clouds differ by an unknown value $\Delta\mathcal{N}$, the measurement has a maximum sensitivity to the gradient determined by $\Delta g/g > \Delta\mathcal{N}/\mathcal{N}$, where $\mathcal{N}$ is the average number of atoms in the cloud.

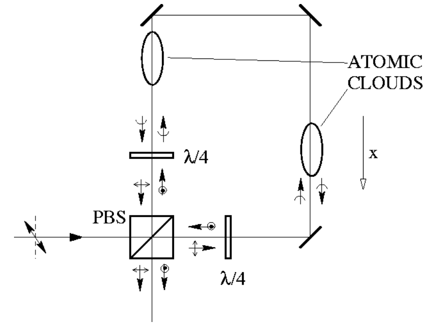

FIG. 2. Measurement scheme for measurement of the gravity field gradient. Two clouds of three-level atoms interact with two counterpropagating waves having the opposite circular polarizations. The gravity field gradient changes atomic velocity and this results in the change of the phase of the electromagnetic waves.

It should be emphasized here that if the number of atoms in each cloud is different, but known, the sensitivity limitation is removed. The scheme gives information about both the acceleration of gravity and its gradient, which may be separated by a filtering procedure. The numbers of atoms in both clouds may be obtained after the interaction via a separate measurement.

III. EQUATIONS OF MOTION

A. Interaction Hamiltonian

Let us consider an atom that moves in the gravity field and interacts with two counterpropagating light beams. The kinetic and potential energies of the atom are

$$\mathcal{T} = \frac{mv^2}{2}, \quad V = mgx, \tag{1}$$

where v is the projection of the atomic velocity on the direction of light propagation (we consider a single dimensional model), m is the mass of the atom, and g is the gravity acceleration. For simplicity we assume that the acceleration does not depend on the atomic coordinate.

The interaction energy of a Λ-configuration atom and two counterpropagating light waves can be presented in slowly varying amplitude and phase approximations as

$$V_1 = \hbar\omega_0 \frac{v}{c}(\hat{\sigma}_{b+b+} - \hat{\sigma}_{b-b-}), \tag{2}$$

$$V_2 = -\hbar\Omega_+\hat{\sigma}_{b+a} - \hbar\Omega_-\hat{\sigma}_{b-a} - \text{adjoint}, \tag{3}$$

where $\hat{\sigma}_{ij} = |i\rangle\langle j|$ is an atomic operator, c is the speed of light in the vacuum, $\Omega_\pm$ are the Rabi frequencies of the two optical fields, and $\Omega_\pm = \wp_\pm E_\pm/\hbar$, $E_\pm$ is the electric field amplitude. The interaction energy V_1 results from the linear Doppler effect that leads to the two-photon detuning $\delta = \omega_0 v/c$ of the electromagnetic fields from the corresponding atomic transitions (see Fig. 1).

Finally, energy of the atom interacting with the fields may be presented in the slowly varying amplitude and phase approximation as

$$V_3 = \hbar \Delta \hat{\sigma}_{aa}, \tag{4}$$

where Δ is the velocity independent single photon detuning of the fields from the corresponding atomic transition. Again the carrier frequencies of the optical fields are assumed to be the same.

To describe the evolution of the system it is convenient to introduce the Lagrange function

$$\mathcal{L} = T - V - V_1 - V_2 - V_3, \tag{5}$$

and find the generalized momentum of the atom

$$P = \frac{\partial \mathcal{L}}{\partial v} = mv - \frac{\hbar \omega_0}{c}(\hat{\sigma}_{b+b+} - \hat{\sigma}_{b-b-}). \tag{6}$$

The Hamiltonian of the system is

$$H = vP - \mathcal{L} = \frac{1}{2m}\left[P + \frac{\hbar\omega_0}{c}(\hat{\sigma}_{b+b+} - \hat{\sigma}_{b-b-})\right]^2 + V + V_2 + V_3. \tag{7}$$

The same result may be obtained if we start from the Hamiltonian [13]

$$\tilde{H} = \frac{P^2}{2m} + mgx + \hbar\Delta\hat{\sigma}_{aa} - (\hbar\Omega_+ \hat{\sigma}_{b+a} e^{ik_+x} + \hbar\Omega_- \hat{\sigma}_{b-a} e^{-ik_-x} + \text{adjoint}), \tag{8}$$

where k_+ and k_- are the wave vectors of the light beams with $k_+ = k_- = k$, P is a generalized momentum of the atom.

To remove the dependence of the interaction energy on the atomic coordinate we introduce a unitary transformation

$$U = \exp[ikx(\hat{\sigma}_{b+b+} - \hat{\sigma}_{b-b-})]. \tag{9}$$

Then the evolution of the transformed state vector $|\Psi\rangle = U^{-1}|\tilde{\Psi}\rangle$ is described by the Hamiltonian (7) $H = U^{-1}\tilde{H}U$. The kinematic atomic momentum may be introduced as $mv = U^{-1}PU = P + \hbar k(\hat{\sigma}_{b+b+} - \hat{\sigma}_{b-b-})$.

B. Hamiltonian equations

Using Eq. (7) we derive equation of motion for the atom

$$\dot{P} = -mg, \tag{10}$$

which has an obvious solution

$$P(t) = P(0) - mgt. \tag{11}$$

This allows us to reduce the Hamiltonian that describes the behavior of atomic internal degrees of freedom

$$H_a = \frac{\hbar k}{m}[P(0) - mgt](\hat{\sigma}_{b+b+} - \hat{\sigma}_{b-b-}) + \hbar\left(\Delta - \frac{\hbar k^2}{2m}\right)\hat{\sigma}_{aa} - \hbar(\Omega_+\hat{\sigma}_{b+a} + \Omega_-\hat{\sigma}_{b-a} + \text{adjoint}), \tag{12}$$

where we have used the normalization condition $\hat{\sigma}_{b+b+} + \hat{\sigma}_{b-b-} + \hat{\sigma}_{aa} = 1$.

From the above Hamiltonian, the time-dependent equations can be generated:

$$\dot{\hat{\sigma}}_{b\pm b\pm} = i(\Omega_\pm \hat{\sigma}_{b\pm a} - \Omega_\pm^\dagger \hat{\sigma}_{ab\pm}), \tag{13}$$

$$\dot{\hat{\sigma}}_{ab\pm} = i(\tilde{\Delta} \mp \delta)\hat{\sigma}_{ab\pm} + i\Omega_\pm(\hat{\sigma}_{aa} - \hat{\sigma}_{b\pm b\pm}) - i\Omega_\mp \hat{\sigma}_{b\mp b\pm}, \tag{14}$$

$$\dot{\hat{\sigma}}_{b-b+} = -2i\delta\hat{\sigma}_{b-b+} + i\Omega_+\hat{\sigma}_{b-a} - i\Omega_-^\dagger \hat{\sigma}_{ab+}, \tag{15}$$

where

$$\tilde{\Delta} = \Delta - \frac{\hbar k^2}{2m}, \qquad \delta = \frac{k}{m}[P(0) - mgt].$$

Therefore, the motion of atoms in the gravitational field results in a time-dependent two-photon detuning of the corresponding atomic transitions driven by the electromagnetic fields. If the gravitational field is small enough or if the carrier frequencies of the electromagnetic fields are properly adjusted, one may consider δ as a slowly varying function of time. The two-photon detuning results in a change of the index of refraction for the electromagnetic fields to be measured experimentally. To describe such a measurement properly, we have to introduce decay terms into the system that does not follow from the Hamiltonian.

C. Propagation problem

Let us consider the interaction of the electromagnetic waves with a cloud of atoms. The stationary propagations of the right and left circular polarized electric field components through the moving atomic cloud are described by the Maxwell equations in the slowly varying amplitude and phase approximation

$$\frac{d}{dx}E_\pm(x) = \pm\frac{2i\pi\omega_0}{c}\wp_\pm N\sigma_{ab\pm}(x), \tag{16}$$

where N is the atomic number density, $\wp_\pm$ are the dipole moments of the respective transitions, and $\sigma_{ab\pm}$ are the c-number analogs of the atomic lowering operators $\hat{\sigma}_{ab\pm} = |a\rangle\langle b_\pm|$. Analytic expressions for $\sigma_{ab\pm}$ can be obtained from the stationary solution of the c-number Bloch equations for the atomic populations [cf. (13)]

$$\dot{\sigma}_{b-b-} = -\gamma_{0r}(\sigma_{b-b-} - \sigma_{b+b+}) + \gamma_r\sigma_{aa} - i(\Omega_-^* \sigma_{ab-} - \text{c.c.}),$$

$$\dot{\sigma}_{b+b+} = \gamma_{0r}(\sigma_{b-b-} - \sigma_{b+b+}) + \gamma_r \sigma_{a\,a} - i(\Omega_+^* \sigma_{ab+} - \text{c.c.}),$$

and polarizations [cf. (14,15)]

$$\dot{\sigma}_{ab\pm} = -\Gamma_{ab\pm}\sigma_{ab\pm} - i\Omega_\pm(\sigma_{b\pm b\pm} - \sigma_{aa}) - i\Omega_\mp \sigma_{b\mp b\pm}, \tag{17}$$

$$\dot{\sigma}_{b-b+} = -\Gamma_{b-b+}\sigma_{b-b+} - i\Omega_-^* \sigma_{ab_+} + i\Omega_+ \sigma_{b-a}, \tag{18}$$

where the generalized decay terms are defined as

$$\Gamma_{ab\pm} \equiv \gamma + \frac{\gamma_{0r}}{2} - i(\tilde{\Delta} \mp \delta), \tag{19}$$

$$\Gamma_{b-b+} \equiv \gamma_0 + \gamma_{0r} + 2i\delta. \tag{20}$$

γ_r is the radiative linewidth of the transition $|a\rangle \to |b_\pm\rangle$, γ is the homogeneous transverse linewidth of the optical transition $|a\rangle \to |b_\pm\rangle$, γ_0 is the transverse coherence decay between the ground levels, and γ_{0r} is the incoherent population exchange rate between the ground levels. We have disregarded Langevin noise forces in Eqs. (17) and (18) associated with spontaneous emission and collisional decay processes, since it was shown in Ref. [17] that these atomic noise types have a negligible effect on the sensitivity of a magnetometer that works by using the same principles.

We calculate the stationary solutions of the Bloch equations by considering only the lowest order in γ_0, γ_{0r}, and δ. We also assume, for the sake of simplicity, that $\tilde{\Delta} = 0$. We find

$$\sigma_{ab_\pm} = \frac{i\Omega_\pm(\gamma_0|\Omega_\mp|^2 + \gamma_{0r}|\Omega_\pm|^2)}{|\Omega|^2(2\gamma[2\gamma_{0r} + \gamma_0) + |\Omega|^2]} \pm \frac{2\delta\Omega_\pm|\Omega_\mp|^2}{|\Omega|^2[2\gamma(2\gamma_{0r} + \gamma_0) + |\Omega|^2]}, \tag{21}$$

where $|\Omega|^2 = |\Omega_-|^2 + |\Omega_+|^2$. Usually the coherence decay between the ground levels dominates the population exchange. However, in our case this rate describes the initial preparation of atomic beam and $\gamma_0 \approx \gamma_{0r}$.

We consider the spatial evolution of amplitudes and phases of the complex Rabi frequencies separately, $\Omega_\pm = |\Omega_\pm| e^{i\phi_\pm}$. The intensities of the two fields are attenuated in the same way

$$\frac{d}{dx}|\Omega_\pm|^2 = \mp \kappa \frac{\gamma_0 \gamma_r |\Omega_\pm|^2}{6\gamma_0\gamma + |\Omega|^2}, \tag{22}$$

where $\kappa = (3/4\pi)N\lambda^2$.

Assuming that $|\Omega(x)|^2 \gg 6\gamma\gamma_0$ and $|\Omega_+(0)|^2 = |\Omega_-(L)|^2 = |\Omega_0|^2/2$, $|\Omega_0|^2 = |\Omega_+|^2_{in} + |\Omega_-|^2_{in}$, we solve Eqs. (22),

$$|\Omega_\pm(x)|^2 = \frac{|\Omega_0|^2}{2}\left[\pm\kappa \frac{\gamma_0\gamma_r}{|\Omega_0|^2}\left(\frac{L}{2} - x\right) + \sqrt{\kappa^2 \frac{\gamma_0^2\gamma_r^2}{|\Omega_0|^4}\left(\frac{L}{2} - x\right)^2 - \kappa L \frac{\gamma_0\gamma_r}{|\Omega_0|^2} + 1}\,\right]. \tag{23}$$

Therefore, steady-state transmission of the light having high enough power through the slowly moving medium can be characterized by the parameter

$$\eta = \frac{W_{out}}{W_{in}} = 1 - \frac{3}{4\pi}\lambda^2 NL \frac{\gamma_0\gamma_r}{|\Omega_0|^2}, \tag{24}$$

where W_{out} and W_{in} are the powers of linearly polarized light before and after the interaction, λ is the wavelength of light, N is the atomic density, and L is the interaction length. The approximation $|\Omega(L)|^2 \gg 6\gamma\gamma_0$ sets an upper limit for the losses, such that $1 \geq \eta \gg 6\gamma\gamma_0/|\Omega(0)|^2$.

Similarly we find the phase equations

$$\frac{d}{dx}\phi_\pm = \frac{\kappa\gamma_r}{2}\frac{\delta}{6\gamma_0\gamma + |\Omega|^2}. \tag{25}$$

The polarization rotation is determined by the relative phase at the output of the system $\phi = [\phi_+(L) - \phi_-(0)]/2$. Assuming that $\phi_+(0) = \phi_-(L) = 0$ we get, from Eqs. (25), $\phi_+(L) = -\phi_-(0)$ and

$$\phi = \frac{\kappa}{2}\int_0^L \frac{\gamma_r\delta}{|\Omega|^2}dx = -\frac{\delta}{2\gamma_0}\ln\left[1 - \kappa L \frac{\gamma_0\gamma_r}{|\Omega_0|^2}\right]. \tag{26}$$

Using Eq. (24) we present the solution in the form

$$\phi = -\frac{\omega_0}{2\gamma_0}\frac{P(0) - mgt}{mc}\ln\eta, \tag{27}$$

where ω_0 is the carrier frequency of light. Both parameters η and ϕ can be measured simultaneously. It is worth noting that the rotation angle does not depend on γ_0 for very small γ_0.

IV. MEASUREMENT SENSITIVITY

A. Measurement strategy

A direct measurement of the polarization rotation ϕ gives the information about the acceleration, as well as about the initial generalized momentum $P(0)$. The initial momentum is uncertain because the initial atomic velocity and the photon number in the fields are uncertain. It is possible to avoid the influence of this uncertainty via a certain filtering procedure [20].

The difference photocurrent is

$$I_- \sim i(E_+^\dagger E_- - E_-^\dagger E_+). \tag{28}$$

We present field E as a sum of an expectation value part and a quantum fluctuation part, $E=\langle E\rangle+e$,

$$e=\int_0^\infty\sqrt{\frac{\hbar\nu}{\mathcal{A}c}}a_\nu e^{-i\nu t}d\nu. \tag{29}$$

Here $\langle E\rangle$ is the expectation value of the field, $\mathcal{A}$ is the effective cross-sectional area of the beam, and a_ν is the annihilation operator, whose commutation relations are

$$[a_\nu,a_{\nu'}]=0,\quad [a_\nu,a_{\nu'}^\dagger]=\delta(\nu-\nu'), \tag{30}$$

where $\nu=\omega_0+\omega$, with side-band frequencies ω. An effective side-band frequency range is determined by the spectral properties of the medium interacting with the radiation and by the measuring device, and is usually much less than the carrier frequency $\omega\ll\omega_0$. Then

$$e\simeq e^{-i\omega_0 t}\sqrt{\frac{\hbar\omega_0}{\mathcal{A}c}}\int_{-\infty}^\infty a(\omega)e^{-i\omega t}d\omega. \tag{31}$$

We assume that (1) the quantum fluctuation part is much less than the mean amplitude of the fields and (2) the atomic acceleration may be written as $g=g_0+\Delta g$, where g_0 is an expectation value and Δg is the unknown signal. The difference current may be presented as a sum of the signal and noise parts $I_-=I_S+I_{N1}+I_{N2}$, where

$$I_S\sim 2|\langle E_+\rangle||\langle E_-\rangle|\frac{\omega_0}{\gamma_0}\frac{\Delta g t}{c}\ln\eta, \tag{32}$$

$$I_{N1}\sim 2|\langle E_+\rangle||\langle E_-\rangle|\frac{\omega_0}{\gamma_0}\frac{P(0)}{mc}\ln\eta, \tag{33}$$

$$I_{N2}\sim i|\langle E_-\rangle|(e_+^\dagger-e_+)+i|\langle E_+\rangle|(e_--e_-^\dagger). \tag{34}$$

The photocurrent is a classical entity [20]. Therefore, we may apply a filtering procedure to it even after it has been recorded. The measurement sensitivity depends on this procedure

$$\tilde{I}_-=\int_0^T G(t)I_-(t)dt, \tag{35}$$

or on the appropriate choice of the filter function $G(t)$. $\tilde{I}_-$ is not a function, but a number. The filtering procedure generally may be understood as a number inferred from the recorded data, which contains the maximum information about the signal and, at the same time, the minimum information about the noise. For example, in the case of a known signal shape and white noise, the shape of the optimum filter function coincides with the shape of the signal. A signal that changes linearly in time may be detected with the best sensitivity if one measures the slope of the data curve.

Let us find the numbers that correspond to the signal and noise determined by Eqs. (32)–(34):

$$\tilde{I}_S\sim 2|\langle E_+\rangle||\langle E_-\rangle|\frac{\omega_0}{\gamma_0}\frac{\Delta g}{c}\ln\eta\int_0^T tG(t)dt,$$

$$\tilde{I}_{N1}\sim 2|\langle E_+\rangle||\langle E_-\rangle|\frac{\omega_0}{\gamma_0}\frac{P(0)}{mc}\ln\eta\int_0^T G(t)dt,$$

$$\tilde{I}_{N2}\sim i\int_0^T G(t)[|\langle E_-\rangle|(e_+^\dagger-e_+)+|\langle E_+\rangle|(e_--e_-^\dagger)]dt.$$

Because the momentum $P(0)$ may be large and not necessarily quantum limited, we restrict ourselves to the class of filter functions such that

$$\int_0^T G(t)dt=0. \tag{36}$$

In this case $\tilde{I}_-$ does not depend on the initial atomic momentum uncertainty, $\tilde{I}_{N1}=0$.

The acceleration measurement sensitivity, i.e., the minimum detectable signal $\tilde{I}_S(\Delta g_{min})$ is determined by the noise level $(\langle\tilde{I}_{N_2}^2\rangle)^{1/2}$. To find the optimum filtering procedure we maximize the ratio of these values

$$\frac{S}{N}=\frac{\tilde{I}_S}{\sqrt{\langle\tilde{I}_{N2}^2\rangle}}. \tag{37}$$

To simplify Eq. (37) we note that the averaged light power is

$$\langle W_\pm\rangle=\frac{\mathcal{A}c}{2\pi}|\langle E_\pm\rangle|^2, \tag{38}$$

so the average photon number emitted by the laser during time T is

$$n_\pm=\frac{\langle W_\pm\rangle T}{\hbar\omega_0}. \tag{39}$$

For light in the coherent state we derive from Eq. (31) an expression for two nonzero field moments:

$$\langle e_\pm(t)e_\pm^\dagger(t')\rangle=\frac{2\pi\hbar\omega_0}{\mathcal{A}c}\delta(t-t'). \tag{40}$$

It is convenient to present $|\Omega(0)|$ and η in terms of the total photon number $n_{in}=n_++n_-$:

$$|\Omega(0)|^2=\frac{3\lambda^2\gamma_r}{8\pi\mathcal{A}T}n_{in}, \tag{41}$$

$$\eta=1-2\frac{\mathcal{N}}{n_{in}}\gamma_0 T, \tag{42}$$

where $\mathcal{N}=\mathcal{A}LN$ is the total number of atoms interacting with light, L is the length of the interaction region, and $\mathcal{A}$ is the cross-sectional area of the light beam (we assume that the area coincides with the size of the atomic cloud). The minimal value of the decay rate of the atomic coherence may be estimated as

$$2\gamma_0 \approx \frac{1}{T}. \tag{43}$$

We now can find

$$\widetilde{I}_S = \left(\frac{2\pi\hbar\omega_0}{\mathcal{A}c}\right) n_{out}\frac{\omega_0}{c}\Delta g T^2 \ln \eta \left[\frac{2}{T^2}\int_0^T tG(t)dt\right] \tag{44}$$

and

$$\langle \widetilde{I}^2_{N2}\rangle = \left(\frac{2\pi\hbar\omega_0}{\mathcal{A}c}\right)^2 n_{out}\left[\frac{1}{T}\int_0^T G^2(t)dt\right]. \tag{45}$$

The ratio (37) transforms to

$$\frac{S}{N} = \frac{\omega_0}{c}\Delta g T^2 \sqrt{\mathcal{N}}\left[\frac{\eta}{1-\eta}|\ln \eta|^2\right]^{1/2} \frac{\frac{2}{T^2}\int_0^T tG(t)dt}{\left(\frac{1}{T}\int_0^T G^2(t)dt\right)^{1/2}}.$$

Let us choose the filter function as $G(t) = t/T - 1/2$. Then condition (36) is fulfilled and Eq. (46) is given by

$$\frac{S}{N} = \frac{\omega_0}{c}\Delta g T^2 \sqrt{\frac{\mathcal{N}}{3}}\left[\frac{\eta}{1-\eta}|\ln \eta|^2\right]^{1/2}.$$

The term in brackets has a maximum about unity for $\eta \approx 0.2$, which means that there is a condition for optimum photon number $n_{in} \simeq \mathcal{N}$. Hence, the maximum of Eq. (46) is

$$\left(\frac{S}{N}\right)_{max} \approx \frac{2\pi\Delta g T^2}{\sqrt{3}\lambda}\sqrt{\mathcal{N}}, \tag{46}$$

which can be further improved by a better choice of the filter function. The minimum detectable signal Δg_{min} can be found from Eq. (46) by writing $(S/N)_{max}(\Delta g_{min}) = 1$.

It is worth noting that Δg_{min} is $\sqrt{3}$ times worse than that of a conventional atom interferometer operating at the shot-noise limit [6,7]. If the initial momentum spread P_0 is small, the optimum filter function is $G(t) = t/T$ and then Δg_{min} is $2/\sqrt{3}$ times better than that of a conventional atom interferometer. This is one of the main results of our paper. We should stress here that this result was obtained without the direct consideration of the quantum projection noise.

The maximum sensitivity (46) depends on the coherence decay rate γ_0, which is chosen to obey to condition (43). Depending on the particular atomic level structure and the experimental setup expression (43) might have a different numeric coefficient, which will result in change of the numeric coefficient in Eq. (46).

B. The role of the coherence decay rate

In the above consideration we have introduced a coherence decay rate γ_0, which is determined by the interaction time of the laser and the atoms. This phenomenological rate shows that atoms initially are prepared not to be in the dark state. One half of atoms should be pumped into the dark state by the interaction with light. This optical pumping results in the absorption that is taken into account by $2\gamma_0 \approx 1/T$.

If the atoms are initially prepared in the dark state, no absorption occurs. The signal and noise currents are then modified:

$$I_S \sim 2|\langle E_+\rangle||\langle E_-\rangle|\omega_0 T\frac{\mathcal{N}}{n_{in}}\frac{gt}{c}, \tag{47}$$

$$I_{N1} \sim 2|\langle E_+\rangle||\langle E_-\rangle|\omega_0 T\frac{\mathcal{N}}{n_{in}}\frac{P(0)}{mc}, \tag{48}$$

$$I_{N2} \sim i|\langle E_-\rangle|(e_+^\dagger - e_+) + i|\langle E_+\rangle|(e_- - e_-^\dagger). \tag{49}$$

The corresponding ratio (37) is

$$\frac{S}{N} = \frac{\omega_0}{c}\Delta g T^2 \sqrt{\mathcal{N}}\sqrt{\frac{\mathcal{N}}{n_{in}}}\frac{\frac{2}{T^2}\int_0^T tG(t)dt}{\left(\frac{1}{T}\int_0^T G^2(t)dt\right)^{1/2}}.$$

Using the same filter function $G(t) = t/T - 1/2$, the above equation transforms into

$$\frac{S}{N} = \frac{2\pi g T^2}{\sqrt{3}\lambda}\sqrt{\mathcal{N}}\sqrt{\frac{\mathcal{N}}{n_{in}}}. \tag{50}$$

There are not any restrictions on n_{in} now because there is no absorption. The sensitivity increases as the atomic number (not as a square root of the atomic number) because all the atoms are in the same state.

According to Eq. (50), it may appear that the maximum sensitivity is achieved for a single photon scattered from the atomic cloud. However, our analysis is based on the assumption that $n_{in} \gg 1$ and on the steady-state solution of the problem.

C. Measurements of atomic velocity

The measurement described here is a quantum nondemolition measurement with respect to the generalized atomic momentum P, Eq. (6). The measurement scheme also gives precise information about the initial velocity of atoms without changing it. In fact, the atomic velocity is not changed by the interaction if the final level population is the same as the initial level population [14].

Let us consider performing a measurement on the velocity $v(0)$ of an atomic cloud. If the atoms are initially prepared in $|b_+\rangle$ state the generalized momentum is determined by the initial atomic velocity and by the constant value of photon recoil momentum $P = mv(0) - \hbar k$ [$\hat{\sigma}_{b+b+}(0) = 1$]. According to our calculation this initial value of the momentum stays unchanged in time if there is no spontaneous emission.

The measurement sensitivity of $v(0)$ is hindered by the shot-noise fluctuations as well as by the back action of the electromagnetic waves due to the atomic nonlinearity. The atomic nonlinearity depends on the two-photon detuning δ, which is proportional to the average generalized atomic momentum $\langle P\rangle$. This back action may be compensated by a constant two-photon detuning δ_0 such that $\delta_0=-\delta=-\hbar k\langle P(0)\rangle/m$. In a real experiment δ_0 can be introduced as a frequency difference between the pump and probe beams.

The signal and noise for atomic velocity measurements are

$$I_S\sim 2|\langle E_+\rangle||\langle E_-\rangle|\frac{\omega_0}{\gamma_0}\frac{v(0)}{c}\ln\eta, \tag{51}$$

$$I_N\sim i|\langle E_-\rangle|(e_+^\dagger-e_+)+i|\langle E_+\rangle|(e_- - e_-^\dagger), \tag{52}$$

where we assume that $|\langle E_+\rangle|\simeq|\langle E_-\rangle|$.

Keeping in mind that the optimum filter function is a constant now, ratio (37) has a form

$$\frac{S}{N}=\frac{4\pi v(0)T}{\lambda}\sqrt{\mathcal{N}}\left[\frac{\eta}{1-\eta}|\ln\eta|^2\right]^{1/2},$$

which has maximum

$$\frac{S}{N}\approx\frac{4\pi v(0)T}{\lambda}\sqrt{\mathcal{N}}. \tag{53}$$

D. Role of finite time duration of the light pulses

The measurement scheme discussed above is based on the atomic coherence and requires an optically thick atomic medium for better sensitivity. Under such conditions the influence of the dipole gradient forces resulting from a finite duration of the probe light pulses might become important [21]. It is known that the ponderomotive force acting on a two-level atom is enhanced significantly in coherent media [22]. We show, however, that for the case of a three-level medium such forces are small.

To find the gradient force we use the Hamiltonian

$$H=\frac{P^2}{2m}+\frac{\hbar k}{m}P(\hat\sigma_{b+b+}-\hat\sigma_{b-b-})+\hbar\left(\Delta-\frac{\hbar k^2}{2m}\right)\hat\sigma_{aa} -\hbar[\Omega_+(t-x/v_g)\hat\sigma_{b+a}+\Omega_-(t+x/v_g)\hat\sigma_{b-a}+\text{adjoint}], \tag{54}$$

where v_g is the group velocity, the values of Rabi frequencies of the counterpropagating fields have finite envelops and, therefore, depend on coordinate x. We assume that in our system $v_g\gg P/m$, which is not always the case for an electromagnetically induced-transparency (EIT) medium [22].

The equation of motion for the atoms may be found from Eq. (54),

$$\dot P=\hbar\frac{\partial}{\partial x}[\Omega_+(t-x/v_g)\hat\sigma_{b+a}+\Omega_-(t+x/v_g)\hat\sigma_{b-a}+\text{adjoint}]. \tag{55}$$

It is easy to see from Eqs. (21) and (55) that if the system adiabatically follows the dark state determined by the coherence population trapping, the gradient term of the force acting on the atoms is small. Estimations show that this term is proportional to δ^2. Generally, if the interaction in the system occurs without populating the excited state of the atom, the gradient ponderomotive force vanishes. This is true for the single-photon resonant interaction of Λ atoms and light, considered above, as well as for the off-resonant interaction considered in the following section.

E. Discussion of the polarization measurement scheme

An increase of the atomic beam size during the interrogation time does not significantly influence the signal-to-noise ratio. Let us consider the case when the laser beam has a cross-sectional area $\mathcal{A}=$const that exceed the atomic beam area at any moment of time. The cross-sectional area of the atomic cloud increases as $\mathcal{A}_b=(\mathcal{A}_0^{1/2}+v_Tt)^2$, where v_T is the average thermal velocity of atoms. The interaction length, which is given by the length of the atomic cloud, increases as well: $L=L_0+v_Tt$. The laser beam may be considered as two fields, one of which interacts with the atomic cloud and another does not. The effective phase shift of the combined fields may be estimated as $\phi=\tilde\phi(\mathcal{A}_b/\mathcal{A})\sim\mathcal{N}/\mathcal{A}$, where $\tilde\phi$ is the phase shift for the light that interacts with the atoms. Therefore, once the atomic cloud is entirely overlapped by the laser beam the atomic spread in space does not change the signal, and the signal is determined by the total number of atoms and the area of the laser beam. The same is valid for the absorption of light if the system is in the EIT regime (the absorption is small). Therefore, the shot noise remains unchanged.

Let us now discuss the restrictions of the scheme. First of all, the absorption of the system should be small. The condition for sustaining coherent population trapping requires $\eta|\Omega_0|^2\gg\gamma_r\gamma_0$. For the optimum field power this expression may be rewritten as

$$0.2\frac{3}{8\pi}\lambda^2LN\gg 1. \tag{56}$$

This condition simply poses a restriction on the atomic density. It shows that the medium should be optically thick ($3\lambda^2LN/8\pi$ shows the number of Beer's absorption lengths).

The absorption problem may be solved by an appropriate preparation of the initial atomic state. The coherence decay rate, introduced phenomenologically, serves only the situation when atoms enter the interaction region without any coherence and the population is distributed between their ground states with equal probabilities. If atoms are initially optically pumped, say, into $|b_+\rangle$ state, and the field E_- is switched before E_+, the spontaneous emission may be suppressed significantly if $|\Omega|^2T_s/\gamma_r\gg 1$, where $|\Omega|$ is the maximum Rabi frequency (we assume that $\gamma_r>|\Omega|$) and T_s is the switching-on-and-off-time ($T>2T_s$).

It is interesting to mention here that condition (56) also serves as the condition that measurement time T should be long enough to prepare the atoms in the "dark state" (if not initially established). In fact, the preparation time for a coherent population trapping, which is $\sim\gamma/|\Omega|^2$ (we assumed $|\Omega|\ll\gamma$), should be much less than $T\approx 1/\gamma_0$. Hence, the steady state may be formed for time scales such that $|\Omega(0)|^2 T \gg \gamma_r$, or $n_{in} \gg 4\pi\mathcal{A}/3\lambda^2 \gg 1$.

Another condition is that the detuning from the two-photon resonance should be kept within the width of the resonance $\eta|\Omega_0|^2 \gg |\delta|\gamma_r$. This condition is stronger than the previous one because δ might be much bigger than γ_0. For example, in the case of atomic fountain, $|\delta|/\gamma_0 \simeq 8\pi L/\lambda \gg 1$. This condition can be substantially relaxed if one tracks the major Doppler frequency shift by constantly sweeping the frequencies of the two circularly polarized waves depending on the average atomic cloud velocities.

Finally, there is a condition that restricts the measurement time from below: $T|\Omega_0|^2 \gg \gamma_r \sqrt{3\lambda^2 LN/8\pi}$ [23]. This condition follows from the solution of the propagation problem in optically thick medium and can be easily implemented experimentally.

It is clear that the scheme is most useful when atoms are first prepared in the dark state. Otherwise, its usefulness is limited in practice for the state-of-the-art experiments. For example, one could not operate the laser intensity at its optimal level while the two-photon Rabi frequency is large enough to cover the residual thermal velocity spread of the atoms. On the one hand, these difficulties may be avoided by using very cold atoms (e.g., Bose-Einstain condensate). On the other hand, both conditions above can be satisfied if the lasers are pulsed. For example, one may utilize the same $\pi/2-\pi-\pi/2$ pulse sequence in the atom interferometry scheme [6]. The measurement of a change in photon number of the pulses due to the interaction gives information about the total number of atoms in the atomic cloud, as well as about the acceleration. The properties of the scheme are discussed below.

V. NONDESTRUCTIVE OPTICAL DETECTION OF ULTRACOLD ATOMS FOR ATOMIC INTERFEROMETRY

We propose to utilize the Raman optical pulses used in light-pulse atomic interferometry for retrieving information about the number of atoms participating in the interaction as well as about interferometer's signal itself. The measurement technique has several advantages.

(1) The balanced measurement is possible, which may reduce the common-mode noise.

(2) The change in the photon number of the pulses results from the interaction with cold atoms only. Background atoms at room temperature do not interact with the fields because they are far-off resonance. This allows operating the atom interferometer with a relatively high vapor pressure without the large interfering background.

(3) As in the previous cw scheme, the differential acceleration may be obtained with a single light signal measurement.

For the case of moderately cold atomic clouds the residual Doppler broadening of the ground-state transition is about 50 kHz, which requires high laser intensity (large photon number in the case of continuous wave lasers) to achieve a significant interaction with the majority of atoms. For the pulsed lasers we may maintain a high laser intensity and small photon numbers, at the same time.

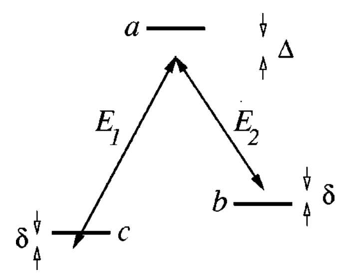

FIG. 3. Lambda scheme. A three-level atom interacts with two counterpropagating waves E_1 and E_2. The atomic motion results in detuning δ of the fields from the corresponding atomic transitions in the atom's frame of reference.

Atomic fountain devices usually utilize long-lived ground-state hyperfine transitions to obtain narrow interference fringes. It is convenient to describe these experiments by using atoms in a Λ-type energy level configuration. We consider a Λ-type atom interacting with two counterpropagating coherent electromagnetic waves (Fig. 3). The waves are far detuned from the corresponding one-photon atomic transitions and are nearly resonant with a two-photon atomic transition. While the excited state $|a\rangle$ has a short lifetime, the ground states $|b\rangle$ and $|c\rangle$ have a long lifetime. Under these circumstances the spontaneous emission from the excited state may be greatly reduced and the three-level configuration may be transformed by a unitary elimination of the excited state to a two-level system coupled by an effective two-photon Raman field.

Because in such a Raman-type configuration the probability of spontaneous emission is rather small, any population transfer from state $|c\rangle$ to state $|b\rangle$ and vice versa is accompanied by an absorption or an emission of a photon in one electromagnetic field, and emission or absorption, respectively, of a photon in the other electromagnetic field. Using this connection between the photon number and the atomic state population we may retrieve the information about the atomic number, using the light.

The main problem here is that the sensitivity of such a measurement *decreases* with increasing photon number, while the sensitivity of atomic fountain experiments *increases* with the photon number, much like the above described polarization rotation measurements. We show, however, that quite a good sensitivity of the measurement may be achieved under some optimum conditions.

A full discussion of the light-pulse atom interferometer has been given in Ref. [6] for atoms with the Λ-type energy level configuration (Fig. 3). Briefly, a $\pi/2$-π-$\pi/2$ pulse sequence is applied to coherently divide, deflect, and recombine an atomic wave packet. All the atoms are initially prepared in state $|b\rangle$. The counter propagating electromagnetic waves E_1 and E_2 create a $\pi/2$ Raman pulse that prepares atoms in a coherent superposition of the ground states and splits them into two wave packets in space. Subsequently, delayed by time T, π pulse exchanges populations of ground states $|b\rangle$ and $|c\rangle$. The atomic clouds meet again in space after time T due to the π pulse. Finally, a time-delayed $\pi/2$ pulse causes the interference between atomic wave packets. This interference may be detected via a measurement of the number of atoms in state $|c\rangle$, for example. In what follows we show that one can retrieve the interferometry information by directly measuring these light pulses.

To describe an interaction of a three-level Λ atom we use the Hamiltonian in slow amplitude and phase approximation:

$$H/\hbar = \Delta\hat{\sigma}_a + \delta(\hat{\sigma}_b - \hat{\sigma}_c) - [|a\rangle\langle c|\Omega_1 + |a\rangle\langle b|\Omega_2 + \text{adjoint}], \tag{57}$$

$\Omega_1 = \wp_1 E_1/\hbar$ and $\Omega_2 = \wp_2 E_2/\hbar$ are the Rabi frequencies for the electromagnetic fields, E_1 and E_2 are envelope functions of the electromagnetic fields interacting with the atom, $\hat{\sigma}_a = |a\rangle\langle a|$, $\hat{\sigma}_c = |c\rangle\langle c|$, $\wp_1$ and $\wp_2$ are the dipole moments of transitions $|a\rangle \to |c\rangle$ and $|a\rangle \to |b\rangle$, $\Delta = (\omega_{ab} + \omega_{ac} - \omega_1 - \omega_2)/2$ is the single-photon detuning, $\delta = (\omega_{ab} - \omega_{ac} - \omega_1 + \omega_2)/2$ is the two-photon detuning.

We proceed by removing state $|a\rangle$ via canonical transformation $\widetilde{H} = \exp(S)H\exp(-S)$. Under the condition $\Delta \gg \delta$,

$$S = |a\rangle\langle c|\frac{\Omega_1}{\Delta} + |a\rangle\langle b|\frac{\Omega_2}{\Delta} - \text{adjoint}. \tag{58}$$

Then the Hamiltonian transforms to

$$\widetilde{H}/\hbar = \left(\delta - \frac{|\Omega_1|^2 - |\Omega_2|^2}{\Delta}\right)(\hat{\sigma}_b - \hat{\sigma}_c) + |b\rangle\langle c|\frac{\Omega_2^\dagger \Omega_1}{\Delta} + |c\rangle\langle b|\frac{\Omega_1^\dagger \Omega_2}{\Delta}. \tag{59}$$

Keeping in mind that the Rabi frequencies of the fields may be presented as $\Omega = \xi\hat{a}$ ($\Omega^\dagger = \xi^* \hat{a}^\dagger$), where ξ is a dimensional parameter and $\hat{a}$ ($\hat{a}^\dagger$) is annihilation (creation) operator, we derive by using Eq. (59)

$$\dot{\hat{n}}_1 = \dot{\hat{\sigma}}_c = -\dot{\hat{\sigma}}_b = -\dot{\hat{n}}_2, \tag{60}$$

where $\hat{n}_{1,2} = \hat{a}_{1,2}^\dagger \hat{a}_{1,2}$ is the photon number in the light pulses. Relation (60) means that a change in atomic population is always accompanied by the same change in the photon number of the electromagnetic waves interacting with the atom.

In the case of atomic accelerometer, fields E_1 and E_2 are counterpropagating. The two-photon detuning is determined by the Doppler effect due to the motion of the atoms,

$$\delta(t) = \frac{\hbar\omega_0}{mc}P(t), \tag{61}$$

where $P = mv - \hbar\omega_0(\hat{\sigma}_b - \hat{\sigma}_c)/c$ is again the generalized atomic momentum that obeys to the Newtonian equation

$$\dot{P} = F_g, \tag{62}$$

v is the velocity of the atom, F_g is a gravitational force.

Let us now consider the transformation when the sequence of $\pi/2$-π-$\pi/2$ pulses is applied to the atom. We present the atomic wave function as $|\psi(t)\rangle = B(t)|b\rangle + C(t)|c\rangle$ ($|B|^2 + |C|^2 = 1$) and assume that $B(0) = 1$ and that the durations of the pulses ($\tau/2$-τ-$\tau/2$) are much less than the time intervals between the pulses (T).

The evolution of the wave function is described by the Schrödinger equation

$$\frac{\partial}{\partial t}|\psi(t)\rangle = -\frac{i}{\hbar}\widetilde{H}|\psi(t)\rangle, \tag{63}$$

which results in set

$$\dot{B} = -i\left(\delta - \frac{|\Omega_1|^2 - |\Omega_2|^2}{\Delta}\right)B - i\frac{\Omega_2^\dagger \Omega_1}{\Delta}C, \tag{64}$$

$$\dot{C} = i\left(\delta - \frac{|\Omega_1|^2 - |\Omega_2|^2}{\Delta}\right)C - i\frac{\Omega_1^\dagger \Omega_2}{\Delta}B. \tag{65}$$

For the sake of simplicity we assume that the optical Raman pulses are very short and classical, such that for the pulse acting from time $t = 0$ to $t = \tau$ we are able to exchange $\delta(t) \approx \delta(\tau/2)$. Then, during the interaction with the light, the evolution of the system is described by

$$B(t) = B(0)\left(\cos(\widetilde{\Omega}t) - i\frac{\widetilde{\delta}}{\widetilde{\Omega}}\sin(\widetilde{\Omega}t)\right) - iC(0)\frac{\Omega_2^* \Omega_1}{\widetilde{\Omega}\Delta}\sin(\widetilde{\Omega}t), \tag{66}$$

$$C(t) = C(0)\left(\cos(\widetilde{\Omega}t) + i\frac{\widetilde{\delta}}{\widetilde{\Omega}}\sin(\widetilde{\Omega}t)\right) - iB(0)\frac{\Omega_1^* \Omega_2}{\widetilde{\Omega}\Delta}\sin(\widetilde{\Omega}t), \tag{67}$$

where

$$\widetilde{\delta} = \delta - \frac{|\Omega_1|^2 - |\Omega_2|^2}{\Delta}, \quad \widetilde{\Omega}^2 = \widetilde{\delta}^2 + \frac{|\Omega_1|^2|\Omega_2|^2}{\Delta^2}.$$

Free evolution of the system is described by expressions

$$B(t) = B(0)\exp\left[-i\int_0^t \delta(t)dt\right], \tag{68}$$

$$C(t) = C(0)\exp\left[i\int_0^t \delta(t)dt\right]. \tag{69}$$

It is easy to see now that the first $\pi/2$ pulse prepares atoms in the state determined by coefficients [6]

$$B_1 = \frac{1}{\sqrt{2}}, \quad C_1 = -\frac{i}{\sqrt{2}}. \tag{70}$$

The subsequent π pulse exchanges the state populations according to relations

$$B_2 = -\frac{1}{\sqrt{2}} e^{i\phi_1}, \quad C_2 = -\frac{i}{\sqrt{2}} e^{-i\phi_1}. \tag{71}$$

Finally, the second $\pi/2$ pulse results in the interference of the atomic states

$$B_3 = -\cos(\phi_2 - \phi_1), \quad C_3 = \sin(\phi_2 - \phi_1). \tag{72}$$

Here $\phi_i \ll 1$ stands for the signal shift of atomic wave function

$$\phi_1 = \int_0^T \delta(t)\, dt, \quad \phi_2 = \int_T^{2T} \delta(t)\, dt. \tag{73}$$

In the case of a constant accelerating force acting on atoms $F_g = mg$, the signal phase shift $\phi_2 - \phi_1 = \Delta g T^2 \omega_0 / c$ depends neither on the average ac-Stark shift nor on the initial uncertainty of the generalized atomic momentum $P(0)$. The independence on the ac-Stark shift may be easily understood if we recall that the sequence of $\pi/2$-π-$\pi/2$ pulses is equivalent to a single 2π pulse. It is easy to see that for such a pulse only terms with quadratic ac-Stark shift are present in the final population.

Let us consider an atomic cloud consisting of $\mathcal{N}$ atoms. Then, if initially the atoms are in state $|b\rangle$, field E_2 of the first $\pi/2$ Raman pulse loses $\mathcal{N}/2$ photons after the interaction, according to Eq. (60). Field E_1, in turn, gains $\mathcal{N}/2$ photons. Therefore, a measurement of the difference of the photon number in the E_1 and E_2 fields gives us the information about the atomic number. The sensitivity of the measurement is determined by the photon shot noise and the projection uncertainty of the final atomic state $(|b\rangle - i|c\rangle)/2^{1/2}$ and may be described by the ratio

$$\frac{S}{N} = \frac{\mathcal{N}}{\sqrt{n + 2\mathcal{N}}}, \tag{74}$$

where n is the total number of photons in both E_1 and E_2 pulses. In the denominator of the ratio, Eq. (74), the first term corresponds to the photon shot noise, while the second term corresponds to the atomic projection noise.

The same type of measurement can be applied for retrieving the information about acceleration signal. Detecting the photon number in the second $\pi/2$ pulse, it can be shown that the sensitivity for the acceleration measurement is determined by the ratio

$$\frac{S}{N} = \frac{\omega_0}{c} \Delta g T^2 \frac{2\mathcal{N}}{\sqrt{n + 2\mathcal{N}}}. \tag{75}$$

Indeed, one of the circularly polarized light pulses gains and the other looses $\mathcal{N}/2 - \mathcal{N}\sin(\phi_2 - \phi_1)$ photons. The atomic cloud in the initial state, $(|b\rangle - i|c\rangle)/2^{1/2}$, has an uncertainty in the population difference $(|b\rangle\langle b| - |c\rangle\langle c|)$ equal to $\mathcal{N}^{1/2}$. This uncertainty is transferred to the uncertainty in the number of photons. The photon shot noise is equal to $(n/2)^{1/2}$ for each polarization. Combining those terms we derive Eq. (75).

The number of photons should exceed $\mathcal{N}$ in the above derivation. Therefore, the sensitivity of this scheme is $\sim \sqrt{n/\mathcal{N}}$ times less than that of the quantum projection noise limited scheme. Nevertheless, the measurement scheme might be interesting because the achievable sensitivity of such a measurement is not a strong function of the number of photons used. Let us estimate the minimum photon number from the condition

$$\frac{\Omega_1 \Omega_2}{\Delta} \tau \approx \frac{3n}{16\pi} \frac{\gamma_r}{\Delta} \frac{\lambda^2}{\mathcal{A}} = \pi, \tag{76}$$

where $\mathcal{A}$ is the cross-sectional area of the laser beam. This would require the total photon number on the order of $n \approx 10^{10}$ for typical state-of-the-art experimental conditions. The total number of atoms is generally much less, $\mathcal{N} \approx 10^7$. Therefore, this technique is more useful with larger atomic densities and colder atomic temperatures, as can be achieved with the Bose-Einstein condensates.

The sensitive detection of photon number of a short pulse is not an easy task itself. However, a specially robust technique proposed and realized for the measurements of observable parameters of optical solitons [24] seems to be feasible for achieving shot-noise limited measurements of the photon number, phase, frequency, and position of a light pulse.

Finally, we should mention here that the intermediate π pulse may be used for checking the ‘‘quality’’ of the atomic state preparation by the first $\pi/2$ pulse. The absorption of both E_1 and E_2 fields should be absent in π pulse if the preparation is perfect and the atomic levels are equally populated.

VI. CONCLUSION

In conclusion, we propose a method for measuring the acceleration of gravity and its gradient, using atomic spectroscopy. The collective motion of an atom cloud causes a polarization rotation that can be measured to retrieve information about this motion. An optimum filtering results in a measurement sensitivity comparable to that of the existing method of light-pulse atom interferometry. We have also shown that the measurement of absorption of light pulses used in light-pulse atom interferometry also provides information about the atomic acceleration as well as the number of the atoms participating in the interaction. While the optimal sensitivity in this scheme requires a high atomic density, it offers an interesting approach that may be advantageous in other applications.

ACKNOWLEDGMENTS

This research was carried out by the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration. A.B.M. gratefully acknowledges stimulating and helpful discussions with F. Narducci, V. A. Sautenkov, and L. J. Wang.

[1] B. Mashhoon and D. Theiss, Phys. Rev. Lett. **49**, 1542 (1982).
[2] B. Mashhoon, H. Paik, and C. Will, Phys. Rev. D **39**, 2825 (1989).
[3] B. Venema, P. Majumder, S. Lamoreaux, B. Heckel, and E. Fortson, Phys. Rev. Lett. **68**, 135 (1992).
[4] T. Damour and A. Polyakov, Nucl. Phys. B **423**, 532 (1994).
[5] N. Sneeuw, R. Rummel, and J. Muller, Class. Quantum Grav. **58**, A113 (1996).
[6] M. Kasevich and S. Chu, Appl. Phys. B: Photophys. Laser Chem. **B54**, 321 (1992).
[7] J.M. McGuirk, G.T. Foster, J.B. Fixler, M.J. Snadden, and M. Kasevich, Phys. Rev. A **65**, 033608 (2002).
[8] G. Santarelli, P. Laurent, P. Lemonde, A. Clairon, A.G. Mann, S. Chang, A.N. Luiten, and S. Salmon, Phys. Rev. Lett. **82**, 4619 (1999).
[9] D.S. Weiss, B.C. Young, and S. Chu, Appl. Phys. B: Lasers Opt. **B59**, 217 (1994).
[10] M.A. Kasevich, Science **298**, 1363 (2002).
[11] M.R. Andrews, M.-O. Mewes, N.J. van Druten, D.S. Durfee, D.M. Kurn, and W. Ketterle, Science **273**, 84 (1996).
[12] S.B. Cahn, A. Kumarakrishnan, U. Shim, T. Sleator, P.R. Berman, and B. Dubetsky, Phys. Rev. Lett. **79**, 784 (1997).
[13] T. Sleator and M. Wilkens, Phys. Rev. A **48**, 3286 (1993).
[14] A.B. Matsko, Phys. Lett. A **275**, 20 (2000).
[15] J. von Neumann, *Mathematical Foundations of Quantum Mechanics* (Princeton University Press, Princeton, 1955).
[16] D. Budker, V. Yashchuk, and M. Zolotorev, Phys. Rev. Lett. **81**, 5788 (1998).
[17] M. Fleischhauer and M.O. Scully, Phys. Rev. A **49**, 1973 (1994).
[18] J.M. McGuirk, G.T. Foster, J.B. Fixler, and M. Kasevich, Opt. Lett. **26**, 364 (2001).
[19] In the case of nondegenerate Λ energy level scheme, the detector signals are modulated at the difference frequency. The polarization rotation phase 2ϕ can be recovered through demodulation.
[20] V.B. Braginsky, M.L. Gorodetsky, F.Ya. Khalili, A.B. Matsko, K.S. Thorne, and S.P. Vyatchanin, Phys. Rev. D **67**, 082001 (2003).
[21] L. J. Wang (private communications).
[22] S.E. Harris, Phys. Rev. Lett. **85**, 4032 (2000).
[23] M.D. Lukin, M. Fleischhauer, A.S. Zibrov, H.G. Robinson, V.L. Velichansky, L. Hollberg, and M.O. Scully, Phys. Rev. Lett. **79**, 2959 (1997).
[24] H.A. Haus, *Electromagnetic Noise and Quantum Optical Measurements* (Springer-Verlag, Berlin, 2000).